Worksheet for Sections 8.4 and 8.6

1. Where applicable in (a) – (c), carry out the necessary division (as in (i) on p. 530) in preparation for integration. If division is not needed, give a reason why it is not needed.

   (a) $\dfrac{x^2}{x^2 - x + 1}$ (b) $\dfrac{x - 1}{x^2 - x + 1}$ (c) $\dfrac{x^4 + 4}{x^2 + 1}$

2. In the following, write the rational function as a combination of factors like those appearing in (4) and/or (5) of Section 8.4, without finding the numerical values of the constants $A_1, ..., B_1, ..., C_1, ...$

   (a) $\dfrac{6x^2 - 4x + 1}{(x - 2)^3}$ (b) $\dfrac{x^3 + 2x^2 + 3x + 4}{x^4 - 16}$

3. (a) Show that $\dfrac{x^2}{(x^2 - 4)^2} = \dfrac{1}{4}\left(\dfrac{1}{x - 2} + \dfrac{1}{x + 2}\right)^2$.

   (b) Use partial fractions to solve $\displaystyle\int \frac{x^2}{(x^2 - 4)^2}\, dx$.

   (c) Could the integral in (b) be solved by trigonometric substitution? Explain your answer.

   (d) Find $\displaystyle\int \frac{\sqrt{x + 4}}{x^2}\, dx$. (*Hint:* Let $u = \sqrt{x + 4}$, and then use the result in part (b)!)

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4. (a) In a complete sentence, tell why there needs to be an *even* number of subintervals in Simpson's Rule.

   (b) Does there need to be an even number of subintervals in the Trapezoidal Rule? Explain your answer in a complete sentence.

5. Suppose f is continuous and positive on the interval $[x_{k-1}, x_{k+1}]$ and let $x_{k-1} < x_k < x_{k+1}$.

   (a) Draw the trapezoid above the interval $[x_{k-1}, x_k]$, with vertices $(x_{k-1}, 0)$, $(x_k, 0)$, $(x_k, f(x_k))$, and $(x_{k-1}, f(x_{k-1}))$. Then find the area A of the trapezoid.

   (b) Using the result of (a), find the sum A_1 of the areas of two trapezoids, the one trapezoid above the interval $[x_{k-1}, x_k]$ and the other trapezoid above the interval $[x_k, x_{k+1}]$. Draw a picture for the two trapezoids.

   (c) Using the formula derived in (b) as a guide, explain why the 2's appear inside the brackets of the Trapezoidal Rule ((2) in the section). Why is there no 2 accompanying $f(x_0)$ or $f(x_n)$?